The Language of Math: Part of the Informational Discourse Continuum

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Abstract

Children with Language Impairment (LI) exhibit difficulty with basic information processing skills that relate to encoding, storing, recalling, organizing, and producing language (Reimann, Gut, Frischknecht, & Grob, 2013). These language-learning problems may impact their ability to understand and use discourse in social and academic settings. One form of informational discourse that is not often discussed, but is critical for success in academic settings, is mathematical discourse. This paper addresses ways in which SLPs may improve students' abilities to profit from instruction in math.

Children with language impairment (LI) exhibit difficulty with basic information processing skills that relate to encoding, storing, recalling, organizing, and producing language (Reimann, Gut, Frischknecht, & Grob, 2013). These language-learning problems may impact their ability to understand and use discourse in social and academic settings. Discourse has been defined as the verbal interchange of ideas and concepts, and as a means for organizing knowledge, thoughts, ideas and experiences using language (Graesser, Millis, & Zwaan, 1997; Kintsch, 1998). Discourse probably exists along a continuum with classroom discourse at one end, narrative discourse in the middle, and informational discourse at the other end.

Classroom discourse is communication that takes place in school settings, and it contains its own inherent set of interactional rules and decontextualized language (Cazden, 1988, 2001; Peets, 2009). The majority of classroom discourse consists of exchanges in which teachers initiate a topic and ask a question, students answer the question, and teachers assess the accuracy of the response. When a student wishes to participate in classroom discourse, he or she must compete for a turn by interrupting or raising a hand. Once acknowledged, the student must either make a contribution to the topic by adding or clarifying information, or by posing a question.

Students with LI have been shown to interrupt less often than students developing typically and are less likely to initiate conversations in the classroom (Craig & Evans, 1993). In addition, students with LI ask fewer questions and often fail to realize when they have misunderstood information (Brinton, Fujiki, & Sonnenberg, 1988; Dollaghan, 1987; Donahue, 1984). Deficits in these critical aspects of classroom discourse have the potential to interfere with the ability to profit from instruction that takes place in other discourse genres, including narrative and informational discourse.
Narrative discourse in academic settings involves the ability to understand and compose stories about real or imagined experiences or events by including common story elements (setting, events, endings) in well-structured stories that contain key, relevant details (CCSS; Common Core State Standards Initiative, 2010). Children with LI experience difficulty with narrative discourse in a number of ways. They are less likely to answer literal or inferential questions about stories (Bishop & Adams, 1992; Gillam, Fargo, & Robertson, 2009; Wright & Newhoff, 2001), they tell shorter, incomplete stories that may contain grammatical errors (Fey, Catts, Proctor-Williams, Tomblin, & Zhang, 2004; Gillam & Johnston, 1992; McFadden & Gillam, 1996; Newman & McGregor, 2006; Roth & Spekman, 1986) and they use less complex sentences and vocabulary than their typically developing peers (Gillam & Johnston, 1992; Greenhalgh & Strong, 2001; Kaderavek & Sulzby, 2000; Scott & Windsor, 2000). Therefore, narrative proficiency is often the target of intervention efforts for children with LI.

Informational discourse is a complex, broad construct that encompasses a wide range of text structures. Lower level structures include sequence, time order/chronology, description, and compare/contrast. More complex text structures that are used to gain a holistic understanding of information include cause-effect and problem-solution. These structures are discussed in detail in Hall-Kenyon and Culatta (this issue). One form of informational discourse that is not often discussed, but is critical for success in academic settings, is mathematical discourse.

Mathematical discourse has only recently been conceptualized as a discourse genre. Sfard (2008) characterized math discourse as a form of communication that is distinguishable by its word use, visual mediators, routines, and narratives (Sfard, 2001, p. 29). Word use involves the use of math vocabulary while visual mediators are symbols to represent the objects that are created during communication surrounding math concepts such as graphs, symbols, words, and diagrams. Routines are defined as discourse specific actions such as answering questions when asked, raising one’s hand to be recognized, or the overarching rules that govern how and when to perform an activity. For example, a student may raise his hand to be recognized before talking, or may answer a question after being asked, or interpret the word rate in one way during math discussions (noun: a measure of frequency) and another way during non-math discussions (verb: assign value to something according to its quality). Finally, the narrative of math discourse refers to the specific spoken or written language used to describe math objects and their relationships as they relate to math-specific word use, visual mediators, and routines. When a math narrative is accurate or true, then it becomes an “endorsed narrative” and encompasses definitions of math terms and concepts such as axioms and theorems. Thus, math discourse is a genre that incorporates knowledge of specialized linguistic constructs along with a wide range of math-specific concepts and linguistic and nonlinguistic structures (Ravid, Dromi, & Kotler, 2013). This domain of discourse is rich and well-specified, and contains a precise set of discourse relations that foster the application of mathematical reasoning (Sfard, 2001).

The ability to solve a math problem depends on one’s understanding of how to combine the components of the problem into a “problem schema” that has been learned. Theoretically, the problem schema is a stored map for solving a certain kind of math problem (much like an expository text structure) that is activated upon hearing, reading, or visualizing the symbols, words, and sentence structures (Hegarty, Mayer, & Monk, 1995). After the problem schema has been activated, the student must implement a specific set of procedures and steps for solving the problem.

There are at least four broad linguistic competencies that may impact the ability to construct, store and retrieve a mathematical problem schema: (a) reading comprehension; (b) knowledge of domain specific vocabulary and syntactic forms that may have slightly different meanings in math than in general language contexts; (c) the ability to shift between how words are used in math-specific and general-language specific contexts; and (d) knowledge of the symbols and formal language used to solve math problems and how these map onto problem schemas. In addition to
these language skills, students must also possess cognitive mechanisms such as working memory and phonological processing skills in order to solve word problems correctly.

Students with LI have been shown to demonstrate difficulties in all of the linguistic and cognitive competencies required for solving mathematical problems. These deficiencies have the potential to impact all levels of math learning including preparatory arithmetic knowledge (e.g., number conservation, classification, seriation), basic mathematical operations (e.g., addition, subtraction, multiplication, division), and higher level mathematical problem solving abilities (Kroesbergen & Van Luit, 2003).

Unfortunately, instructional interventions for students with learning disabilities, many of whom also have concomitant language impairment, have not been found to be very effective (Fuchs & Fuchs, 2014; Hegarty, Mayer, & Monk, 1995). Most math instruction for diverse learners (e.g., those with learning disabilities, language impairment, mild intellectual disability, or reading disorders) addresses basic computational skills (addition, subtraction, multiplication, and division) rather than mathematical problem solving. This may be because studies of instruction that have focused on improving mathematical problem solving in diverse learners have not been shown to be as effective as instructional strategies that focus on basic math abilities (Hegarty et al., 1995; Kroesbergen & Van Luit, 2003). This is partly because problem-solving instruction may not sufficiently address the linguistic and cognitive factors (such as attention control, working memory, and language comprehension and production) that are often associated with learning disorders (Fuchs, Fuchs, & Compton, 2013).

The Language of Math

What can speech-language pathologists (SLPs) do to improve students’ abilities to profit from instruction in math? First, we must have a clear understanding of the linguistic concepts that are included in mathematical discourse that may pose a problem for students with LI. Silliman and Wilkinson (2015, p. 295) provide an excellent review of the vocabulary and syntactic features that are present in math and science discourse that may be problematic for students with language problems. These linguistic skills include but are not limited to knowledge of highly technical, precise vocabulary related to math concepts, knowledge of how words may be “re-designated” from general language contexts to reference specific math concepts (i.e., length, prove, rate), and the ability to understand and process complex and elaborated noun phrases (e.g., Present three trigonometric formulas that could be used to solve the following problem). Further, math concepts and processes are sometimes signified by nominalization (verbs or adjectives that are used as nouns). For example, the word sum may be used as a verb in the sentence “You must sum up the numbers to find the answer” or as a noun in the sentence, “The sum of the problem is 10.” Math also makes use of symbols to express relationships between objects and processes established by relational verbs (e.g., is, be, have, means). For example, a = b means “a is equal to b.” Further, there are many words that have math-specific meanings. For example, the word “mean” in math discourse is defined as the average of a set of numbers divided by the number of numbers in the set. Attempts to explain these concepts to students with language impairment using simple vocabulary and sentence structures may be quite challenging.

Math instruction and math problems often contain complex sentence structures such as relative clauses and passive constructions, which place a heavy burden on already limited information processing capacities (see Alt, Arizmendi & Beal, 2014 and Silliman & Wilkinson, 2015 for a complete review). The following is an example of a simple addition word problem that was taken from a math workbook for second and third graders (Miller, 2014).

“Helen the Hippo and her friends are preparing for Thanksgiving at Helen’s house. Find out how many of each food they have prepared for the party. Dylan the Dog prepared 241 hotdog sticks in a brown bag. His father placed 426 more hotdog sticks in the same brown bag. How many hotdog sticks did Dylan and his father place in the brown bag?” This math problem may
present issues for a child with LI for a number of reasons that have nothing to do with the ability to perform the mathematical calculation of addition. There are at least two Tier 2 vocabulary words (prepare, place) in the problem that may be unfamiliar. These words may be unfamiliar because they are not used frequently in daily, informal language and present discrepancies between concepts that are understood (put = Tier 1 word) and words that are not (preparing = Tier 2) or because they are being used differently in the math problem than in general language contexts (e.g., place usually means where not to put). The student must understand the adverbial phrase, how many of each food and be able to process it in the context of the entire problem. Adverbials have been shown to be difficult for students with LI (Marinelle, 2004). The construction is used twice in the problem and may pose a cognitive as well as a conceptual burden on working memory. In one sentence, there is a complex elaborated noun phrase in the same brown bag that may also pose an undo cognitive load on memory because of its length. Further, the word same is used which may present a conceptual problem for the student if it’s meaning is unclear.

Finally, there is a relative clause in the instructions (that they have prepared). Research has shown that many students with LI demonstrate difficulty comprehending relative clauses (Nippold, Mansfield, Billow, & Tomblin, 2008). In this example, the relative clause/phrase is also accompanied by a Tier 2 word (prepared) that may be unfamiliar to diverse learners.

**SLPs Supporting Math Instruction**

Clearly, language and cognitive processing abilities contribute to how well diverse learners (e.g., LI and English language learners) will profit from instruction in math. So, where do we start? Research suggests that there are at least two aspects of language that SLPs may be able to address in intervention that will support math learning and contribute to what math educators are already doing well. The first is intensive instruction in vocabulary relevant to math. The SLPs already have the knowledge and skills needed to teach vocabulary. The important thing is to be able to identify the critical words that are needed for students to understand the math concepts they will be taught, and the contexts that words will be presented in. The second aspect of language that SLPs may address is understanding and use of various types of complex sentences, such as those containing relative clauses, that are frequently present in math problems.

**Addressing the Vocabulary of Math**

To teach vocabulary to support math instruction, SLPs must be aware of the kinds of words that are important for students to know. There are many websites and materials that contain this kind of information. North Central Educational Service District (NCESD, n.d.) hosts a website that contains organized lists of math words that are arranged by grade level according to the common core curricular standards for each grade. Other well-supported websites for math vocabulary include Mathwords (2014) and one hosted by the Virginia Department of Education (2016). The Virginia website not only lists the words by grade level, but contains the written word, a visual graphic that illustrates the concept, and the numerical expression that accompanies it. The flashcards are available in color for free downloading. Examples of some of the math terms that were included for first grade are, addition, sum, plus, subtraction, difference, minus, number sentence, equation, length, time, and weight.

After the words have been selected, Beck, McKeown, and Kucan (2013) recommend teaching student-friendly definitions and explanations for them. Student friendly definitions should capture the essence of the word and how it is usually used and explain its meaning in everyday language. First, describe the word using Tier 1 words to the extent possible. Tier 1 words are basic vocabulary words that students have likely encountered and learned in their daily lives (Beck et al., 2013). Then, explain how the word is used in everyday language and in this case, how it may also be used in math discourse. For example, the word “rate” may mean the speed that something is traveling, the number of times something has or is occurring, or an amount or price that is set to pay or acquire something. It would be important to talk about and give real world examples for each of
these meanings, and then practice identifying them in authentic math problems. After the word(s) have been described and explained, students might benefit from explaining the word(s) to the clinician and/or a peer.

Examples of student friendly definitions for math concepts may be found on various websites and sources, however we found one to be particularly comprehensive. Math is Fun (2016) is a website that contains student friendly definitions along with examples, graphics, and online games for teaching and practicing mathematics concepts and vocabulary. For example, addition is defined as “bringing two or more numbers (or things) together to make a new total.” Then a graphic is shown to illustrate the concept of addition with an accompanying written explanation, “Here 1 ball is added to 1 ball to make 2 balls.” The number sentence is shown under the explanation and graphic (The number sentence is written as $1 + 1 = 2$), and is followed by a written number sentence, “One plus one equals two.” These examples along with games for practicing concepts and vocabulary related to addition may be found on the Math is Fun (2016) website. The Math is Fun site covers grades kindergarten through high school. We recommend first, going to the home page of Math is Fun (2016) and clicking on the superordinate category named numbers, which will take you to a page that contains basic mathematical vocabulary and concepts for performing addition, subtraction, multiplication and division problems. There, clinicians may explore each of the basic math areas and make decisions about the words that need to be taught. There is also a link to a section called word questions that is found under more about add, subtract, multiply, and divide. There, SLPs can get a sense of the kinds of syntactic frames in which vocabulary concepts and words are typically presented. These sentence structures and linguistic concepts are discussed in the next section and will need to be incorporated into instruction. The pages may also be used for additional practice for students. Other ideas for teaching math language is to use word walls and math vocabulary journals. The K-5 Math Teaching Resources (2016) website contains examples of math terms, visual representations, activities, and teaching ideas that may be used in instruction. Beck et al. (2013) have shown that key features of effective vocabulary instruction include frequent and varied encounters with words presented in activities that encourage deep processing (p. 83). They recommend that students learn 10 words per week with daily follow up activities, so that by the end of the week, each word has been encountered at least 10 times. Importantly, they stress that the activities be varied and illustrate different contexts in which the word may be used.

One activity that may be used to deepen students’ understanding of math discourse terms is to ask them to write narrative essays. As Petersen and Petersen (this issue) discuss, there are many ways in which narrative and expository discourse are similar. In these narrative essays, students are challenged to incorporate mathematical terminology into their stories (Seo, 2015). For example, the student would be given a list of math terms such as intersect, midpoint, distance, end point, and parallel lines and be asked to write a story using them. The student may require support in developing the basic narrative that might say,

Sally and Joe met in their neighborhood to talk about what they wanted to do that afternoon. They wanted to go to the candy store but it was too far. They decided to go to the library. They walked together to the library and found a great book to read together.

The student’s story might be rewritten to say,

Sally and Joe intersected at the midpoint of their neighborhood to talk about what they wanted to do that afternoon. The distance to the candy store was too far so they decided that their endpoint should be the library. They walked in parallel lines to the library and found a great book to read together.

These kinds of activities are engaging, challenging and require deeper levels of processing that goes beyond knowledge of a word’s definition. This notion of robust instruction advocated by Beck and her colleagues is an important part of vocabulary interventions that result in lasting outcomes for students (Beck et al., 2013, p. 83).
**Addressing the Syntax of Math**

In addition to vocabulary, students must be proficient in processing the syntactic frames in which the problems are presented. This applies not only to word problems, but to number sentence problems as well since the symbols represent linguistic and cognitive concepts. Consider the following problem taken from the Math section of the Common Core State Standards for first grade: “Jessica has some crayons. She finds eight more. Now Jessica has 17 crayons. How many did Jessica have before she found more? Write a subtraction sentence, and the addition sentence that helped you subtract.”

The problem contains *before she found more* which is a subordinated adverbial clause. “Write a subtraction sentence and the addition sentence” is a coordinated noun phrase followed by the subject relative clause (that helped you subtract). Schuele and Tolbert (2001) have shown that subject relative clauses produced by typical 3-to-5-year-olds included the obligatory relative marker (e.g., Point to the dog *that* is running) but the 5-to 7-year-old children with LI who participated in their studies omitted 63% of obligatory subject relative markers (e.g., Point to the dog is running). If the *Jessica problem* is representative of the kinds of word problems encountered by typical first graders, it is likely to pose a problem for students with LI, who may not have mastered these and other syntactic skills necessary for understanding it (e.g., subordinated adverbial clauses).

There are a number of resources for SLPs to access to unpack the sentence structures that are used in math discourse. An excellent one is entitled, “Multicausal Utterances Aren’t Just for Big Kids: A Framework for Analysis of Complex Syntax Production in Spoken Language of Preschool- and Early School-Age Children” (Arndt & Schuele, 2013). Their article contains a framework for analyzing complex syntax in spontaneous language for preschool and early school-age children. This comprehensive review of complex syntax, along with a suggested coding system (Schuele, 2009) and supplemental digital content, may be helpful for assisting SLPs in deepening their understanding of the types of complex sentences that play a role in real-world and academic skills. These analyses may be helpful in informing intervention decisions that will lead to functional academic outcomes in math performance (Supplemental Digital Content; Wolters Kluwer, n.d.-, n.d.-b, n.d.-c).

Weiler and Schuele (2014) developed another excellent resource on complex sentences that addresses subordinate conjunction clauses and may be helpful for assessing student’s ability to profit from math instruction. The article is a comprehensive tutorial on the grammatical, lexical, and pragmatic functions served by subordinate conjunction clauses. They include examples of the subordinate conjunction (i.e., after), its meaning-relation (i.e., temporal-time), and an example sentence (i.e., He went to the ice cream parlor after he ate dinner at the new restaurant) for a number of forms. Consider the following math problem taken from the common core curricular standards: “If 18 apples are arranged into 3 equal rows, how many apples will be in each row?” In this problem, the subordinate conjunction *(if)* is associated with the meaning-relation conditional. In order to fully understand the problem, at the very least the student must understand the conditional semantic relations conveyed by the subordinate conjunction *(if)*, to mean “*If* the 18 apples are arranged in 3 equal rows, *then* how many apples will be in the row?”

Kamhi (2014) provides an excellent brief tutorial describing how clinicians might work with students on understanding the meanings and functions that are conveyed by syntactic structures rather than directly teaching the structures. There are also websites that are available for assisting SLPs and their students in learning and practicing specific syntactic skills that are needed for understanding math discourse. One is called No Red Ink (2016) and contains lessons, quizzes, and practice activities for students in grades kindergarten through college. First, you select the grade level, name your class, and then assign work. You may select the type of assignment (diagnostic, practice, or quiz), assign points for the activity, and then select the topics that you or your student needs to practice. For example, for second grade, you may select from 50 topics.
including active and passive voice, building compound and complex sentences, phrases and dependent clauses, and coordinating and subordinating conjunctions to name only a few. A series of lessons on identifying coordinating and subordinating conjunctions (FANBOYS) and their functions is available along with a link to practice lessons on for, and, nor, but, or yet, and so. For example, you may click specific interests and a page with TV shows and movies, Disney characters and superheroes, actors and actresses, etc. comes up. You select the one that you like (e.g., Harry Potter) and click continue. A sentence is shown on the screen and you are asked to highlight the coordinating conjunction (e.g., Hermione Granger has a clarinet lesson this afternoon so she can’t come to the park with us). After practicing, you may take a quiz on the topic. The Wall Street Journal did a piece on this website stating that it is overhauling the way writing is taught in the United States (Chapman, 2015).

Another website that may be used for practice is IXL (2016). The website contains lessons and practice for math, language arts, science and social studies for grades K–12. If, for example, a student needs further practice using conjunctions, you may click on second grade conjunctions. The student is given forced choice sentence completion tasks such as, “Mr. Blanchard has a truck ___ not a tractor”. The student clicks on but or or and clicks “submit.” Written feedback, such as Good job or Fantastic is provided when answers are correct and extensive corrective feedback is given if the answers are incorrect. For example, we selected but rather than so after seeing the sentence, “Someone left the cage open, ___ my pet rabbit got out.” Then, a lesson popped up to explain what a conjunction was and gave examples of how to use each one. The explanations included the following:

- And is used to show that things go together;
- But is used to show that things are different or opposite, or to show a choice;
- So is used to show that something happens because of something else.

At the end of the section, there was an explanation as to why the word so was the best conjunction for the practice sentence because it connects two things to show that something happens because of something else.

**Summary**

Research has shown that there are a number of reasons why children with LI may encounter problems in math. These include, but are not limited to, the ability to manipulate math symbols, the use of working memory to identify and store patterns, and the difficulty of combining complex vocabulary, syntax, and math symbols to solve problems (Alt, Arizmendi, & Beal, 2014). Difficulties with math discourse has the potential to impact the ability to solve math word problems, but also in learning exact arithmetic skills such as retrieving math facts from memory and verbal counting (Nys, Content, & Leybaert, 2013). Therefore, it is important that SLPs plan instructional activities to improve functional math outcomes for students with LI.

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